LON-CAPA
Mathematical Functionality
Gerd Kortemeyer
Virginia Commonwealth University
May 2011
LON-CAPA Mathematics

- Mathematics Output:
  - typesetting
  - graphing

- Generating Mathematics Problems:
  - symbolic math functionality
  - statistics packages

- Mathematics Input:
  - numerical
  - formula evaluation
    - sampling
    - symbolically
    - checking for properties
  - graphical input
  - bubble sheets
  - clickers
Today’s presentation is going to show some very specialized functionality. Because you can does not mean you have to.
Mathematical Output

- Typesetting: LaTeX can be embedded anywhere in the material

\[ x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \]
Mathematical Output

- Editor for non-native \LaTeX speakers
Mathematical Output

- Generated on-the-fly, can vary from student to student.
Mathematical Output

- `<algebra>`-tag to pretty-print the output from computer algebra systems
- Example: `$\text{formula} = "a \times x^5"$`
Mathematical Output

- One-source, multiple target
- Looks good in print
  - Online:
    - The solution is
    \[ x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \]
  - Print (dynamically generated PDF):
    - The solution is
    \[ x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \]
Mathematical Output

- Dynamic Graphing
  - Data-Points
  - Functions
  - Line-Graphics
- Internally uses GNUplot
Mathematical Output

- Data points
Mathematical Output

- Data points
Mathematical Output

- Functions
Mathematical Output

- Line graphics
Generating Mathematics Problems

- LON-CAPA problems include
  - Perl Scripting Environment
  - MAXIMA Computer Algebra System
  - R Statistics Package

- Problems not just randomized, but randomly generated with desired properties
Generating Mathematics Problems

- Direct calls to MAXIMA:
  $result=&cas('maxima',$expression);
- Simple example: use computer algebra system to calculate a reduced fraction

```plaintext
# Construct an Egyptian Fraction that can be represented by three terms with denominators between 3 and 12
@denominators=(&random_permutation(&random(1,1000,1),(3..12)))[0..2];
$egyptian='1/'.(join('+1/',sort{$a<=$b}@denominators)));
$possible="A possible solution is $egyptian";

# Let the CAS figure out the value
$solution=&cas('maxima',$egyptian);

Write 103/165 as an Egyptian Fraction

Submit Answer  Tries 0

Answer for Part 0  A possible solution is 1/3+1/5+1/11
```
Generating Mathematics Problems

- Direct calls to R:
  $result=&cas('R',$expression);
  $results=&cas_hashref('R',$expression);

- Example: generate a distribution with certain properties:

```plaintext
$seed=&random(1,500,1);
$n=&random(15,25,1);
$offset=&random(2,5,0.1);
$slope=&random(0.6,2.5,0.1);
# construct a data set using R
# dump is for debugging, print to screen to see data structure
($data,$dump)=&cas_hashref('R','set.seed($seed);x<-1:$n;w<-1+sqrt(x)/2;data.frame(x=x,y=$offset+$slope*x+rnorm(x)*w);');
@x=&cas_hashref_array($data,'x');
@y=&cas_hashref_array($data,'y');
```
Simplest input: numerical
Sampling – approximate function

```
$amplitude=&random(2.5,3.8,0.1);
$omega=&random(1,3,0.1);
$k=&random(0.2,0.4,0.1);
$phi=&random(0.3,3,0.1);
# Correct function from computer
$function="$amplitude*exp(-$k*x)*sin($omega*x+$phi)";
```

** Gnuplot compatible curve function

$function

** Match the function indicated in black.

f(x) =

** Response: Formula

<table>
<thead>
<tr>
<th>Answer: $function</th>
<th>Sample Points: x@0;1;2;3;4;5;6;7;8;9</th>
</tr>
</thead>
</table>

**
Symbolically: exactly one exact answer (but equivalent forms)
Mathematical Input

- Checking properties
  - Using R:

Provide a list of 3 numbers (separated by commas) that has a mean value of 6.2.

```
x <- c(RESPONSE[1], RESPONSE[2], RESPONSE[3]);
abs(mean(x) - LONCAPALIST[1]) < 0.001
```
Mathematical Input

- Checking properties
  - Using MAXIMA:

Give an example of a function

1. which is orthogonal to

\[ -2 \cdot \cos(5 \cdot x) + 2 \cdot \sin(4 \cdot x) \]

with respect to the scalar product

\[
\langle g \mid h \rangle = \int_{-\pi}^{\pi} g(x) \cdot h(x) \, dx
\]

2. whose norm is 1.

Answer algorithm

```
overlap: integrate((RESPONSE[1])*(LONCAPALIST[1]), x, -%pi, %pi)/%pi;
norm: integrate((RESPONSE[1])*(RESPONSE[1]), x, -%pi, %pi)/%pi;
is(overlap=0 and norm=1);
```
Mathematical Input

- Checking properties
  - Using Perl and MAXIMA:

```perl
# Subroutine that checks if the provided term is indeed an Egyptian Fraction
sub analyze {
  my ($expression)=@_;  
  $expression=~s/\s//gs;  
  $expression=~s/\+?1\%/\,\)/gs;  
  if ($expression=~/\^[/\([^0-9]+\)]+$/) {
  # Format is indeed 1/n+1/m+...
  $last=-1;
  foreach $number (sort { $a<=$b } split(\/,\,,$expression)) {
  # Is a number used twice?
  if ($last==$number) { return(0,1); }
  $last=$number;
  }
  return(0,0);
  }
  return(1,0);
}
```

```perl
# Analyze the format
($formaterror,$doubleerror)=&analyze($submission);
if ($formaterror || $doubleerror) { return 'WRONG_FORMAT'; }
# It is an Egyptian Fraction, is the value correct?
if (&cas('maxima',$submission.'-'.$egyptian.'')) eq '0' {
  return 'EXACT_ANS';
}
return 'INCORRECT';
```
Mathematical Input

Math editor for students
Mathematical Input

At $t=0$ s, a car cruises at a constant positive velocity. Suddenly, a light switches to red. At $t=10$ s, the driver is maximum on the brake. The car then stops in front of the red light for over 2 seconds. Eventually, it drives off, and then again cruises at a constant velocity. The car cannot accelerate with more than 3 m/s$^2$.

Provide a graph of its acceleration as a function of time.

- Graphical input
- using Geogebra
Mathematical Input

- Rule sets

Function Plot Evaluation Rule

<table>
<thead>
<tr>
<th>Index/Name:</th>
<th>Function: Function itself</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial x-value:</td>
<td>Initial x-value label: Type-in value brake</td>
</tr>
<tr>
<td>Final x-value (optional):</td>
<td>Final x-value label (optional): Type-in value stop</td>
</tr>
<tr>
<td>Minimum length for range (optional):</td>
<td>Maximum length for range (optional):</td>
</tr>
<tr>
<td>Relationship: less than</td>
<td></td>
</tr>
<tr>
<td>Percent error:</td>
<td></td>
</tr>
</tbody>
</table>

Function

First Derivative

Second Derivative

Integral

Symbolic, computed, or hard-coded ranges
Mathematical Input

- Problems can also be rendered for bubble sheets
- Every student gets a different version

A crate with a mass of 177.5 kg is suspended from the end of a uniform boom with mass of 88.5 kg. The upper end of the boom is supported by a cable attached to the wall and the lower end by a pivot (marked X) on the same wall. Calculate the tension in the cable.

22. A $2.58 \times 10^3$  B $2.92 \times 10^3$  C $3.29 \times 10^3$
  D $3.72 \times 10^3$  E $4.21 \times 10^3$  F $4.75 \times 10^3$
  G $5.37 \times 10^3$  H $6.07 \times 10^3$
Mathematical Input

Name:

LB271 Fall 2009 Final Exam Version A

1 pt In which scenario does the incline exert a lower normal force on the block?
- A) The left scenario.
- B) The right scenario.
- C) Same in both scenarios.

2 pt In which scenario does the incline exert a lower frictional force on the block?
- A) The left scenario.
- B) The right scenario.
- C) Same in both scenarios.

3 pt By how many decibels does the sound intensity from a point source decrease if you increase the distance to it by a factor of 8?
- A) 12.5
- B) 13.8
- C) 15.6
- D) 17.6

4 pt A particle is located at y = 2.0 mm and has a kinetic energy of 29.5 Joules. What is the maximum x-coordinate the particle could reach? (in mm)
- A) 0.1
- B) 0.7
- C) 1.6
- D) 2.6

5 pt At what fraction of the speed of light (β) does Enterprise see the shuttle approach?
- A) 0.00
- B) 0.30
- C) 0.98
- D) 0.91

6 pt The shuttle has a length of 9 meters when at rest. How long is it in the system of Deep Space 9? (in m)
- A) 1.3
- B) 2.6
- C) 3.7
- D) 5.4

7 pt Captain Picard on the Enterprise takes a 9-minute tea break. How long is this break in the system of Deep Space 9? (in min)
- A) 0.07
- B) 0.13
- C) 0.18
- D) 0.23

8 pt You have two organ pipes of the same length, one closed at both ends, and the other half open. Which one has a lower fundamental frequency?
- A) The closed pipe.
- B) Same.
- C) The half-open pipe.

9 pt In a very simple model of the lower atmosphere, air has a constant density of 1.20 kg/m³. How much would the air pressure change over a height difference of 130 m? (in Pa)
- A) 960
- B) 1110
- C) 1300
- D) 1430

10 pt You have two identical looking spools (same mass, same shape, same size). However, one is hollow, made from iron, the other is solid, made from aluminum. A string is wound around each spool. If you pull on both strings with equal force, which spool is going to have the larger angular acceleration?
- A) Same
- B) Perpendicular to the surface.
- C) Uphill.
- D) None of the above.
A block is being held in place on an incline. The magnitude of the force applied by the hand on the block is the same in the left and the right scenarios.

1. A) The left scenario.  
B) The right scenario.  
C) Same in both scenarios.

2. A) The left scenario.  
B) The right scenario.  
C) Same in both scenarios.

A particle is located at \( x = 5.5 \, \text{mm} \) and has a kinetic energy of 9.8 Joules. What is the minimum x-coordinate the particle could reach? (in mm)

4. A) 1.6  
B) 2.6  
C) 2.7  
D) 2.9

Deep Space Nine sees Enterprise and a shuttle approach from exactly opposite directions with 0.4 \( c \) and 0.4 \( c \) respectively.

5. A) 0.60  
B) 0.47  
C) 0.50  
D) 0.59

A car drives in the forward (positive) direction. It first has a constant speed, then drives into a parking spot, waits for a few moments, then drives out again backwards. Which one of the acceleration graphs could describe this scenario?

9. A) Scenario A  
B) Scenario B  
C) Scenario C  
D) Scenario D  
E) None of the above.

You have two identical looking spoons (same mass, same shape, same size). However, one is hollow, made from iron, the other is solid, made from aluminum. A string is wound around each spoon. If you pull on both strings with equal forces, which spoon is going to have the larger angular acceleration?

13. A) The hollow spoon  
B) The solid spoon  
C) Same

A block is sliding uphill as shown. What is the direction of the frictional force on the block?

11. A) Perpendicular to the surface.  
B) Downhill.  
C) Uphill.  
D) None of the above.
Mathematical Input

CODE - AACHDA
LB 271 - Introductory Physics Lecture
Version A

Name:

LB271 Fac
Mathematical Input

Numerical Clicker
in Lecture

Resistance

Current through a resistor

+5V

R = 200 Ω

-3V

What is the current in milliampere?
LON-CAPA can evaluate clicker data after lecture
Grading (msu_8p96131ebae7b47b8msul1 ss08lbs272)

Current Resource: Mon, Mar 10th

Part: 0 score Type: numerical

Specify a file containing the clicker information for this resource.

Choose File: MonMar10thA.csv

Type: i>clicker

- Award points just for participation
- Correctness determined from response by course personnel
- Correctness determined from response with clicker ID(s)

Percentage points for correct solution: 100
Percentage points for incorrect solution: 60

Upload File
Mathematical Input

• i>clicker2 integrated in LON-CAPA
Thank You

Gerd Kortemeyer
Lyman Briggs College
and
Division of Science and Mathematics Education

korte@lite.msu.edu
http://www.lon-capa.org/